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Answers to TA session # 1

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See at the end of this document the complete solution to the problem on futures in the lecture of January 30th, slides 48 and 49.

DURATION

1. A one-year, $100,000 loan carries a market interest rate of 12 percent. The loan requires payment of accrued interest and one-half of the principal at the end of six months. The remaining principal and accrued interest are due at the end of the year.

TO SOLVE THIS PROBLEM WE USE EQUATION (3) IN LECTURES NOTES OF 01.30.07.

a. What will be the cash flows at the end of 6 months and at the end of the year?

Cash flow in 6 months = $100,000 x 0.12 x 0.5 + $50,000 = $56,000 interest and principal.
Cash flow in 1 year = $50,000 x 0.12 x 0.5 + 50,000 = $53,000 interest and principal.

b. What is the present value of each cash flow discounted at the market rate? What is the total present value?

$\$56{,}000 \div (1+R/2)^{2\times 1/2} = \$56{,}000 \div 1.06 = \$52{,}830.19 = PVCF_{1/2}$
$\$53{,}000 \div (1+R/2)^{2\times 1} = \$53{,}000 \div (1.06)^2 = \underline{\$47{,}169.81} = PVCF_1$
$= \$100{,}000.00 = \text{PV Total CF}$

c. What proportion of the total present value of cash flows occurs at the end of 6 months? What proportion occurs at the end of the year?

$\text{Proportion}_{t=1/2}$ = ($52,830.19 ÷ $100,000) x 100 = 52.830 percent.
$\text{Proportion}_{t=1}$ = ($47,169.81 ÷ $100,000) x 100 = 47.169 percent.

d. What is the duration of this loan?

Cash flow in 6 months = $56,000 interest and principal.
Cash flow in 1 year = $53,000 interest and principal.

Time(t)	Cash Flow	DF	CF×DF=PVCF	CF×DF×t
1/2	$56,000	0.943396	$52,830.19	$26,415.10
1	$53,000	0.889996	$47,169.81	$47,169.79
		Denominator =	$100,000.00	$73,584.89 = Numerator

$$D = \frac{\$73,584.89}{\$100,000.00} = 0.735849 \text{ years}$$

The maturity of this loan is 1 year but the duration is 0.7358 years. The principal is recovered in more or less in 9 months. After that time the lender makes profits or return on the loan. Notice that already after 6 months the lender receives 53.49% of the loan.

HEDGING INTEREST RATES: FUTURES

2. Consider the following balance sheet (in millions) for an FI:

Assets		Liabilities	
Duration = 10 years	$950	Duration = 2 years	$860
		Equity	$90

a. What is the FI's duration gap?

Note that from equation (8) lectures notes from 01.30.07, the change in equity (net worth) due to a change in the interest rate is obtained from:

$\Delta E = -\, \mathbf{[D_A - D_L \times L/A]} \times A \times (\Delta R/(1+R))$.

We also defined the duration gap (DGap) as to be equal to $\mathbf{[D_A - D_L \times L/A]}$

The duration gap is 10 - (860/950)(2) = 8.19 years.

b. What is the FI's interest rate risk exposure?

The FI is exposed to interest rate increases. The market value of equity (E) will decrease if interest rates increase. *ΔE will be negative when interest rates increases because DGap is positive*.

When the DGap is positive, *<u>ΔE will be negative</u>* if and only if $(\Delta R/(1+R))$ is positive, that is, when there is an increase in the interest rates.
When the DGap is negative, *<u>ΔE will be negative</u>* if and only if $(\Delta R/(1+R))$ is negative, that is, when there is an decrease in the interest rates.

c. How can the FI use futures and forward contracts to put on a macrohedge?

The FI can hedge its interest rate risk by selling futures or forward contracts.

d. What is the impact on the FI's equity value if the relative change in interest rates is an increase of 1 percent? That is, $\Delta R/(1+R) = 0.01$.

ΔE . - [8.19]×950×(.01) . -\$77.805 millions after using:

$$\Delta E = - [D_A - D_L \times L/A] \times A \times (\Delta R/(1+R))$$

e. Suppose that the FI in part (c) macrohedges using Treasury bond futures that are currently priced at 96. What is the impact on the FI's futures position if the relative change in all interest rates is an increase of 1 percent? That is, $\Delta R/(1+R) = 0.01$. Assume that the deliverable Treasury bond has a duration of nine years.

Keep in mind first that in general with Treasury bonds, the minimum futures-contract size is \$100,000. (For Treasury bills, the minimum futures-contract size is \$1,000,000.)

To answer this question you use equation (11) in lectures 01.30.07 which is:

$$\Delta F = - D_F(N_F \times P_F) \times (\Delta R/(1+R));$$

The lecture notes indicates that such equation gives us the **gain off balance sheet on futures** from selling or buying futures: That is the compensation that the FI will get when the interest rate changes once the FI has entered into a futures contract.

As indicated in the lecture notes, in the above formula, we will have $-N_F$ is you are selling futures contracts and $+N_F$ if you are buying futures contracts. Again, N_F is number of contracts.

The FI will be here selling futures. The impact on the FI's futures position or the gain off balance sheet PER CONTRACT as a result of the increase of the interest rate: $\Delta R/(1+R) = 0.01$ is:

$\Delta F = - 9 \times ((-1) \times 96{,}000) \times (.01) = \$8{,}640$ **<u>per futures contract.</u>**

MATURITY

3. Consumer Bank has $20 million in cash and a $180 million loan portfolio. The assets are funded with demand deposits of $18 million, a $162 million CD and $20 million in equity. The loan portfolio has a maturity of 2 years, earns interest at the annual rate of 7 percent, and is amortized monthly. The bank pays 7 percent annual interest on the CD, but the interest will not be paid until the CD matures at the end of 2 years.

   a. What is the maturity gap for Consumer Bank?

   M_A = [0*$20 + 2*$180]/$200 = 1.80 years
   M_L = [0*$18 + 2*$162]/$180 = 1.80 years
   MGAP = 1.80 – 1.80 = 0 years.

   b. Is Consumer Bank immunized or protected against changes in interest rates? Why or why not?

   It is tempting to conclude that the bank is immunized because the maturity gap is zero. However, the cash flow stream for the loan and the cash flow stream for the CD are different because the loan amortizes monthly and the CD pays annual interest on the CD. Thus any change in interest rates will affect the earning power of the loan more than the interest cost of the CD.

Problem in the lecture (slides 48 and 49):

The bank should sell futures contracts since an increase in interest rates would cause the value of the equity and the futures contracts to decrease.

This is because the Duration Gap is $(6 - 4\times(135/150)) = 2.4$

AGAIN:
When the DGap is positive, *ΔE will be negative* if and only if $(\Delta R/(1+R))$ is positive, that is, when there is an increase in the interest rates.
When the DGap is negative, *ΔE will be negative* if and only if $(\Delta R/(1+R))$ is negative, that is, when there is an decrease in the interest rates.

a. For an increase in the interest rate of 100 basis points, the change in equity on-the-balance sheet position is:

$\Delta E = -[D_A - D_L\times L/A]\times A\times(\Delta R/(1+R))$
$\Delta E = -[2.4]\times\$150m\times(0.01/1.10) = -\$3,272,727.27.$

b. How many contracts are necessary to fully hedge the bank?

If the market value of the underlying 20-year is \$95 per \$100, assuming the interest rate of also of 10%, and the duration is 10.05 as shown on the last page of this chapter solutions. The number of contracts to hedge the bank is:

$$N_F = \frac{(D_A - D_L\times L/A)\times A}{D_F\times P_F} = \frac{(6-4\times 0.9)\$150m}{10.05\times\$95,000} = \frac{\$360,000,000}{\$954,750} = 377.06\,contracts$$

c. Verify that the change in the futures position will offset the change in the cash balance sheet position for a change in market interest rates of plus 100 basis points, and minus 50 basis points.

For an increase in rates of 100 basis points, we know that:

$\Delta E = -\$3,272,727.27.$

Now, with an increase in the interest rate, the value of the Treasury bonds decreases. Because Tree Row Bank engaged in selling futures contracts for hedging, the Exchange Traded Markets will compensate this Bank for the loss in the value of these bonds that were used for hedging. How much will Tree Row Bank be compensated for? This is calculated from:

$$\Delta F = -D_F(N_F\times P_F)\times(\Delta R/(1+R));$$

$$\Delta F = -10.05\times((-377)\times\$95{,}000)\times(0.01/1.10) = \$3{,}272{,}188.64$$

This $3,272,188.64 (the off-balance-sheet gains) does not offset 100% the negative change in equity of $3,272,727.27. This is because this Bank is buying 377 contracts and not 377.06 which were suppose to be the optimal. But we cannot buy 0.06 contract.